A Monte Carlo model of three-dimensional radiative transfer

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1. Introduction

This paper describes a radiative transfer model whose earlier versions have been used in various studies, including Loeb et al. (1997), Di Girolamo et al. (1998), Várnai and Davies (1999), and Várnai (2000). The model is presented in two parts: first, Section 2 briefly describes the model's main features, then Section 3 discusses the model's accuracy.

2. Model description

The basic algorithm of the presented model is very similar to the Monte Carlo techniques described in other studies (e.g., O'Hirok and Gauthier 1998). The model uses the maximum cross section approach, described by Marchuk et al. (1980). While this technique sometimes slows down the simulations (depending on factors such as cloud geometry and the magnitude of cloud density variations), it certainly makes the model's code more simple and easier to understand.

The Monte Carlo approach of simulating the path of a finite number of photons introduces random errors into the results. These errors can be especially large in calculating the spatial distribution of reflectivity values. In order to reduce the noise in reflectivity fields, the model uses a variance-reduction technique described by Marchuk et al. (1980), which has also been discussed by Marshak et al. (1998). Using this technique, absorptivity and reflectivity values are updated not only once during the simulation of each photon, when it leaves the atmosphere, but each time it gets scattered. (Flux values, however, are updated only when photons leave the cloud field.) The reflectivity value of a pixel at point (x,y) is calculated using the equation

$$I(x,y) = \frac{N_{pixels}}{N_{photons}} \sum_{N'} P \cdot E$$

where N_{pixels} is the number of pixels in the scene, $N_{photons}$ is the number of simulated photons, and the summation is over all scattering events (N') that occur inside our pixel. The function P is the scattering phase function describing the probability that a photon would be scattered into the exact view direction (given its original direction before the

scattering), and E is the escape function describing that if the photon was scattered into the exact view direction, what would be the probability that it could emerge from the cloud layer without further scattering or absorption.

The model is set up in a way that it calculates all required quantities (absorption, fluxes, and reflectivities) in a single simulation. It also uses a single simulation to obtain results for both the conservative and absorbing cases.

Finally, one should mention the model's main physical limitations:

- It does not include the effects of polarization,
- It can handle only ice crystals that are randomly oriented,
- It ignores atmospheric refraction.

Since, however, none of the problematic effects are included into the intercomparison project, these limitations do not influence the results' accuracy.

3. Accuracy of results

Since the overall accuracy of model outputs can be best assessed through comparisons with results obtained by other models—which is a central goal of the I3RC project—this paper evaluates only the statistical uncertainties of the results, which can be evaluated by considering this model only.

The accuracy of individual pixel values is estimated by dividing the entire simulation into 100 equal photon batches, and examining how much the partial results vary from one batch to another. The standard deviation of the results for these batches ($\sigma_{batches}$) is used to estimate the statistical uncertainty of the entire simulation (σ_{total}), by considering that the statistical uncertainty decreases with the square root of the number of simulated photons. Thus, since the overall result is based on 100 times more photons than the results from the individual batches,

$$\sigma_{\text{total}} = \frac{\sigma_{\text{batches}}}{10}$$

Since the issue of statistical uncertainties is most critical for reflectivity values (for which it is most difficult to reduce the noise), this section focuses on the statistical uncertainties of the calculated reflectivity fields.

The most straightforward Monte Carlo models calculate reflectivity values based on the number of simulated photons that leave a pixel in directions close to the view angle. Probability theory implies that the uncertainty of the reflectivity values obtained this way is proportional to the square root of the true reflectivity value. Figure 1 shows that the noise-reduction technique described in Section 2 does not change this relationship drastically.

The model's efficiency can be evaluated through the relationship between the number of simulated photons ($N_{photons}$) and the scene-average value of the uncertainty of individual pixel reflectivities (σ):

$$\sigma = \frac{a}{\sqrt{N_{photons}}}$$

where *a* indicates how efficient the model is (smaller *a* values imply less noise, and thus higher efficiency).

In examining the *a* values of the various simulations, the most striking feature is that *a* increases drastically when the Henyey-Greenstein phase function is replaced by the C.1—even

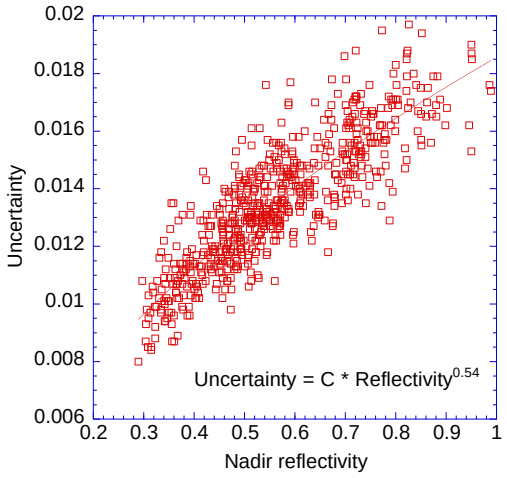

FIG 1. Relationship between the estimated uncertainty and the estimated true value of individual pixel nadir reflectivities. (Radar cloud, C.1 phase function, $\Theta_0=60°$, $\omega_0=1$)

though the scene-averaged reflectivity hardly changes at all (Figure 2.). The increase can be explained through the following argument.

The presented model reduces the uncertainty of a pixel's estimated reflectivity by calculating it based on not the relatively rare events when a photon leaves the pixel in a direction close to the nominal view angle (as the most straightforward Monte Carlo approach does), but based on all scattering events that occur within the pixel (Section 2). This is advantageous, because the more events are used to build a statistics (such as a pixel's average reflectivity value), the smaller the result's uncertainty will be.

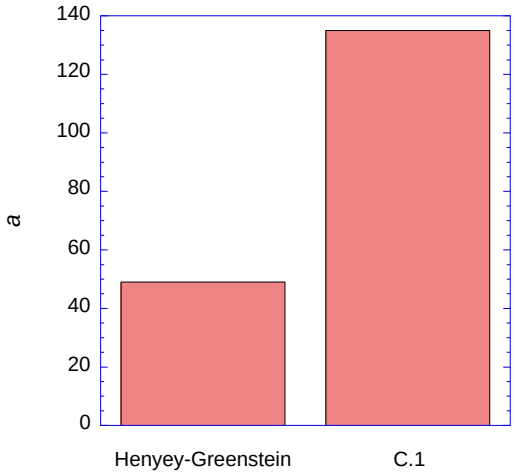

FIG. 2. The efficiency factor *a* (in arbitrary units) for various scattering phase functions. (Radar cloud, $\Theta_0=60°$, $\omega_0=1$)

The gain's main limitation is that, as discussed in Section 2, not all scattering events have equal weight: the weight of each scattering event is proportional to the phase function value $P(\Omega,\Omega')$, where

Ω is the view direction, and Ω' is the photon's original direction before the scattering. For strongly peaked phase functions (such as the C.1), this means that a single scattering event can get a huge weight if the photon's original direction was close to the nominal view angle. In comparison, the influence of all the other scattering events may be negligible, because the phase function value is orders of magnitudes smaller for large scattering angles. Thus the pixel's reflectivity value is determined by the few scattering events in which a photon's original direction is close to the view direction; and building statistics from a few important events implies large uncertainties. Since the Henyey-Greenstein phase function has a much smaller forward scattering peak (Figure 3), this effect poses much weaker limitation to the noise-reduction technique, and, as a result, $a_{HG} < a_{C.1}$.

The results (not shown) also indicate that the efficiency of the noise-reduction technique is much less sensitive to single scattering albedo than to the phase function of cloud particles.

4. Conclusions

The presented model has been developed with the main goal of reducing the human work involved in setting up various experiments. As a result, the main focus was to create a code that is easy to use, flexible, and has a clear, simple structure. While reducing the simulation time was also a goal, it was less in the center of development efforts.

The model uses a noise-reduction technique that is especially effective for relatively smooth phase functions, such as Henyey-Greenstein. In contrast, it requires significantly longer simulations for particles with highly peaked phase functions, such as C.1.

In addition to reducing the simulation noise, the adapted technique also has the advantage of avoiding artificial smoothing effects that could arise if reflectivity fields were calculated based on the number of photons that leave each pixel within a finite angular bin around the view angle.

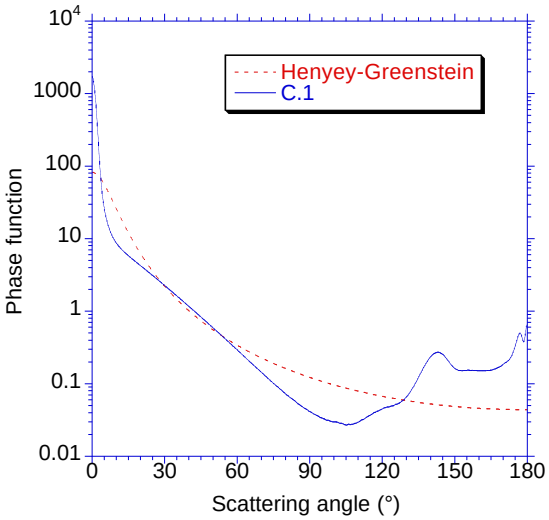

FIG 3. Comparison of the C.1 and Henyey-Greenstein phase functions.

Acknowledgments

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